The Analytics of Information and Uncertainty
Answers to Exercises and Excursions

Chapter 5: Information and Informational Decisions

5.1 Information-Some Conceptional Distinctions

This section contains no exercises.

5.2 Informational Decision Analysis

5.2.1 The use of evidence to revise beliefs

Solution 5.2.1.1. Since at least one of booths #2 and #3 must be empty (and the M.C. knows which one it is), it might appear that his drawing the curtain conveyed no information, and hence that there is no basis for the contestant to change her choice. But such an inference is incorrect. The tabular form that follows represents a convenient procedure for employing Bayes' Theorem to obtain the posterior probabilities implied by any given message m (the message here being "booth #2 is empty").

state of the world	prior prob	likelihood	joint prob	posterior
Prize is in #1	1/3	1/2	1/6	1/3
Prize is in #2	1/3	0	0	0
Prize is in #3	1/3	1	1/3	2/3
			1/2	1

Table 1: Computation of posterior probabilities (after message m)

The next-to-last column represents the column of the joint probability matrix J associated with the particular message received, while the adjoined sum at the bottom of the column is the overall probability of that message. (Once the contestant chose booth #1, there were equal prior chances of the M.C. opening the curtain of either booth #2 or #3.) The last column corresponds to the relevant column of the potential posterior matrix S. Note how the tabular form makes it easy to compute the posterior probabilities . Evidently, the best choice now is booth #3. Intuitively, the M.C.'s action told the contestant nothing about booth #1, her initial choice. But it was a valuable message as to booth #2 versus #3.

Remark 1. Another way to understand this problem is to look at a 100-doors version. There are 100 doors and only one has a prize inside. You pick a door, the M.C. opens 98 empty doors and asks whether you will change. Intuitively, the remaining door is highly likely to have the prize, and most people would choose to change.

Solution 5.2.1.2.

(A) The potential posterior matrix $\Pi = [\pi_{s.m}]$ is

$$\Pi = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$$

where the row is indexed by states and column by messages. Thus, $\pi[s_1|m_1] = 0.9,\ \pi[s_2|m_1] = 0.1$ etc. Next, using Equation (5.2.4) we compute (q_1, q_2), the unconditional probabilities of m_1 and m_2

$$\begin{aligned} \pi_1 &= 0.7 = 0.9q_1 + 0.2q_2 \\ \pi_2 &= 0.3 = 0.1q_1 + 0.8q_2 \end{aligned}$$

to obtain $(q_1, q_2) = (5/7, 2/7)$. Then we calculate the joint distribution $J = [j_{sm}]$ by $j_{sm} = \pi_{s \cdot m} q_m$:

$$J = \begin{bmatrix} 9/14 & 2/35 \\ 1/14 & 8/35 \end{bmatrix}$$

Finally, we can obtain $L = [q_{m \cdot s}]$ by $q_{m \cdot s} = j_{sm}/\pi_s$:

$$L = \begin{bmatrix} 45/49 & 4/49 \\ 5/21 & 16/21 \end{bmatrix}$$

(B) For (i), we need nothing because we can get the marginal, hence conditional distribution from the joint distribution.

For (ii), we need the prior π_s so that we can recover the joint $j_{sm} = q_{m \cdot s}\pi_s$ and then the rest of the distributions.

For (iii), we need q_m so that we can recover the joint $j_{sm} = \pi_{s \cdot m} q_m$ and then the rest of the distributions.

(C)

(i) To get conclusive information we need different states induce different messages. For example,

$$L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

It is not unique; an off-diagonal matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ will also do.

(ii) To get completely useless information, we need that different states induce the same message probability distribution. For example,

$$L = \begin{bmatrix} a & 1-a \\ a & 1-a \end{bmatrix}.$$

for any $a \in [0, 1]$. Obviously, it is also not unique.

(iii) Suppose the prior is π and such an L matrix exists. In particular, m_1 is conclusive and m_2 is uninformative. Let $(q_1, 1 - q_1)$ be the marginal probabilities of the messages. Then it must be that

$$\Pi = \begin{bmatrix} 1 & \pi \\ 0 & 1-\pi \end{bmatrix}$$

The J matrix is then

$$J = \begin{bmatrix} q_1 & \pi(1-q_1) \\ 0 & (1-\pi)(1-q_1) \end{bmatrix}$$

So the L matrix is

$$L = \begin{bmatrix} \frac{q_1}{\pi} & 1-q_1 \\ 0 & 1-q_1 \end{bmatrix}$$

which is not a likelihood matrix for any π as the rows do not add to one. Hence, it is impossible to have exactly two messages of which one is completely informative and the other completely uninformative.

5.2.2 Revision of optimal action and the worth of information

Solution 5.2.2.1. The possible terminal actions are x_1 (do not bet), x_2 (bet on heads), and x_3 (bet on tails). The best action on the basis of your prior information is obviously $x_0 = x_1$. The available message service μ is a sample of size 1, the possible messages being a head or a tail. Following the tabular method of exercise 5.2.1.1, we can compute the posterior probabilities given the message $m =$ "head" as follows:

Since the posterior probabilities are 2/3 for "coin is two-headed" and 1/3 for "coin is fair," the posterior probability of heads, after message $m =$ "head", is $(1) \cdot (2/3) + (0.5) \cdot (1/3) = 5/6$. The best posterior terminal action is therefore x_2 (bet on heads), with expected gain $U(x_2) - U(x_0) = 30 \cdot (5/6) - 50 \cdot (1/6) = 16\frac{2}{3}$. By a similar calculation, the message $m =$ "tails" would lead to exactly

state of the world	prior prob(π_s)	likelihood($q_{m\cdot s}$)	joint prob(j_{sm})	posterior($\pi_{s\cdot m}$)
Coin is two-headed	1/3	1	1/3	2/3
Coin is fair	1/3	1/2	1/6	1/3
Coin is two-tailed	1/3	0	0	0
			1/2	1

Table 2: Computation of posterior probabilities (after message "head")

the same utility gain from the optimal posterior action x_3 (bet on tails). So, by Equation (5.2.8), $16\frac{2}{3}$ is the worth of the message service.

Solution 5.2.2.2.

(A) Being risk neutral means the agent simply minimizes the expected loss. Since

$$E[L(R,P)] = 100(0.04-0.02)0.7 = 1.4$$

$$E[L(A,P] = 0.1\cdot 200(0.06-0.04) + 0.1\cdot 200(0.08-0.04) = 1.2$$

The agent should choose accept.

(B) First we compute the posteriors.

state of the world	prior prob(π_s)	likelihood($q_{m\cdot s}$)	joint prob(j_{sm})	posterior($\pi_{s\cdot m}$)
		m=good, m=defect	m=good, m=defect	m=good, m=defect
Defect rate 0.02	0.7	0.98, 0.02	0.686,0.014	0.709, 14/32
Defect rate 0.04	0.1	0.96, 0.04	0.096,0.04	0.099, 4/32
Defect rate 0.06	0.1	0.94, 0.06	0.094,0.006	0.097, 6/32
Defect rate 0.08	0.1	0.92, 0.08	0.092,0.008	0.095, 8/32
			0.968, 0.032	1, 1

Table 3: Computation of posterior probabilities (after message "good", "defect")

Then we can compute the expected utilities of the two actions under the two posteriors. When m = good, A yields a lower expected loss then R, hence he chooses A, and gets

$$E[L(A,P)] = 200(0.06-0.04)0.097 + 200(0.08-0.04)0.095 = 1.148.$$

When m = "defect" choosing R yields a lower expected loss, where he gets

$$E[L(R,P)] = 100(0.04-0.02)\times\frac{14}{32} = 0.875.$$

Since $q_g = 0.968, q_c = 0.032$, the expected loss for having the sample is

$$EU = 0.968 \times 1.148 + 0.032 \times 0.875 = 1.14.$$

Hence he is willing to pay $1.2 - 1.14 = 0.07$ for a sample.

Solution 5.2.2.3.

(A) For $v(c) = \ln c$, we know that individual 1's demand will be $c_s = \pi_s W/P_s$. With conclusive information, individual 2 will then consume W/p_s for each state s. Thus

$$\begin{aligned} U_2 - U_1 &= \sum_s \pi_s \ln \frac{W_2}{P_s} - \sum_s \pi_s \ln \frac{\pi_s W_1}{P_s} \\ &= \sum_s \pi_s [\ln W_2 - \ln P_s] - \sum_s \pi_s [\ln \pi_s + \ln W_1 - \ln P_s] \\ &= \ln W_2 - \ln W_1 - \sum_s \pi_s \ln \pi_s. \end{aligned} \tag{1}$$

(B) Let K be the agent's willingness to pay for information. Then when $W_2 = W - K$ inserted in (1) we need $U_2 - U_1 = 0$. Thus

$$\ln(W - K) - \ln W = \sum_s \pi_s \ln \pi_s.$$

Take exponential on both sides to obtain

$$\frac{W - K}{W} = \Pi_s \pi_s^{\pi_s}.$$

The fraction of wealth $K^* := K/W$ is then

$$K^* = 1 - \prod_{s=1}^{S} \pi_s^{\pi_s}.$$

(C) Set up a Lagrangian for

$$\min_{\pi_s} \prod_{s=1}^{S} \pi_s^{\pi_s} \text{ s.t. } \sum_s \pi_s = 1$$

to obtain the solution $\pi_s = 1/S$ for all s. Intuitively, the uncertainty is greatest when all states are equally likely.

Solution 5.2.2.4.

(A) Given $\pi_1 = \pi_2 = 1/2$ we can solve for (q_1, q_2):

$$\frac{1}{2} = 0.75q_1 + 0.25q_2$$
$$\frac{1}{2} = 0.25q_1 + 0.75q_2$$

to obtain $q_1 = q_2 = 1/2$. Hence we can solve the J matrix

$$J = \begin{bmatrix} \frac{0.75}{2} & \frac{0.25}{2} \\ \frac{0.25}{2} & \frac{0.75}{2} \end{bmatrix}$$

and then the L matrix

$$L = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

(B) With $v(c) = \sqrt{c}$, it is routine to show that the demand function is given by

$$c_s = \frac{W\pi_s^2/P_s^2}{\frac{\pi_1^2}{P_1} + \frac{\pi_2^2}{P_2}}.$$

Hence the indirect utility function $V(\pi_1, P_1, W)$ is then

$$V(\pi_1, P_1, W) = \pi_1\sqrt{\frac{W\pi_1^2/P_1^2}{\frac{\pi_1^2}{P_1} + \frac{\pi_2^2}{P_2}}} + \pi_2\sqrt{\frac{W\pi_2^2/P_2^2}{\frac{\pi_1^2}{P_1} + \frac{\pi_2^2}{P_2}}}.$$

Now $V(1/2, 1/2, 100) = 10$, and

$$V(0.75, 1/2, 100 - \xi) = 0.75\sqrt{2(100-\xi)}\sqrt{\frac{\pi_1^2}{\pi_1^2 + \pi_2^2}} + 0.25\sqrt{2(100-\xi)}\sqrt{\frac{\pi_2^2}{\pi_1^2 + \pi_2^2}}$$
$$9 + 1 = 10$$

when $\xi = 20$.

Hence the willingness to pay is $\xi = 20$.

5.2.3 More informative versus less informative message services

Solution 5.2.3.1.

(A) With three states we can represent posterior beliefs as in Figure 5.5 (in the text). At the vertex A_s of the triangle, the probability of state s is 1. Therefore the triple (A_1, A_2, A_3) is the perfect information service. Any other information service with three messages can be represented as a triangle which has the prior probability vector in its interior, since the prior is just the message-weighted average of the posteriors.

In Figure 5.5 two information services $\hat{\mu}$ and μ are depicted. Each vertex of a triangle represents the probability vector associated with a particular message. Note that the triangle for μ lies inside the triangle for $\hat{\mu}$. (It is not difficult to confirm that the three posterior probability vectors must be linearly independent unless the three points in the figure lie on a line.) It follows that each posterior $\pi_{\cdot m}$ of message service m is a convex combination of the posteriors of message service $\hat{\mu}$, that is:

$$\Pi = \hat{\Pi} A.$$

Also, for consistency we know that

$$\Pi q = \hat{\Pi} A q = \pi = \hat{\Pi} \hat{q},$$

where q and $\hat{q}$ are the message probability vectors and π is the prior probability vector. But, since the columns of $\hat{\Pi}$ are linearly independent, this matrix is invertible. It follows immediately that $Aq = \hat{q}$ and thus the information service $\hat{\mu}$ is preferred to μ.

(B) With two states and three messages, consider two information services for which the implied posterior beliefs are as follows:

$$\hat{\mu}: \quad \hat{\Pi} = \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix} \text{ and } \hat{q} = (\epsilon, 1-2\epsilon, \epsilon)$$

$$\mu: \quad \Pi = \begin{bmatrix} \frac{5}{6} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{2} & \frac{5}{6} \end{bmatrix} \text{ and } q = (\frac{1}{2} - \epsilon, 2\epsilon, \frac{1}{2} - \epsilon).$$

The posterior probability vector $(5/6, 1/6)$ is a convex combination of $(1, 0)$ and $(1/2, 1/2)$. Similarly $(1/6, 5/6)$ is a convex combination of $(1/2, 1/2)$ and $(0, 1)$. Formally, we have

$$\Pi = \hat{\Pi} A, \qquad \text{where } A = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ \frac{1}{3} & 1 & \frac{1}{3} \\ 0 & 0 & \frac{2}{3} \end{bmatrix}$$

However, it is no longer possible to draw any conclusion about the relative value of the two information services since many different message probabilities are consistent with prior beliefs of $(1/2, 1/2)$. In particular, for all $\epsilon \in [0, 1/2]$ the above data are consistent with such prior beliefs, since in each case

$$\Pi q = \hat{\Pi} \hat{q} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

For ϵ sufficiently close to zero the information service $\hat{\mu}$ updates an individual's prior with very low probability while information service μ updates with probability close to 1. It is therefore necessarily the case that μ is preferred over $\hat{\mu}$ for ϵ sufficiently close to zero. With ϵ close to 1/2, however, the opposite is true. Information service $\hat{\mu}$ is almost perfect while μ has almost no value.

(C) At the end of the answer to (A) we established that if one information triangle lies inside the other all the conditions for a strict ranking are satisfied. With four messages and one quadrilateral inside the other, this is no longer the case.

Solution 5.2.3.2.

(A) Since $\pi_{s \cdot m} = \sum_{\hat{m}} \hat{\pi}_{s \cdot \hat{m}} a_{\hat{m}m}$ for all s, we can express this in vector notation as

$$\pi_{\cdot m} = \sum_{\hat{m}} a_{\hat{m}m} \hat{\pi}_{\cdot \hat{m}}.$$

(B) No. Consider a two state example with both states equally likely. Suppose that μ is conclusive. That is, in state s only m_s is sent. Now let $\hat{\mu}$ be the message service that, when in state s there is a 50% chance of sending m_s and 50% chance of sending an uninformative message m_3. Then clearly $\hat{\mu}$ is not preferred to μ. The equation $\Pi = \hat{\Pi} A$ is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

(C) Since $\hat{\Pi} A = \Pi$, it suffices to show that $Aq = \hat{q}$. To this end, suppose there exists q_1, q_2 s.t. $\Pi q_i = \pi$ for $i = 1, 2$. Then $\hat{\Pi} A q_i = \pi$ for $i = 1, 2$. Suppose $A(q_1 - q_2) \neq 0$. Then $\hat{\Pi}(A(q_1 - q_2)) = \pi - \pi = 0$, which violates the assumption $rank(\hat{\Pi}) = \hat{M}$. Hence $A(q_1 - q_2) = 0$. It then follows from the same reasoning and that $rank(A) = \hat{M}$ that $q_1 = q_2$. Hence q is unique. Since $\hat{\Pi}\hat{q} = \hat{\Pi} A q$, we have $\hat{q} = Aq$.

Solution 5.2.3.3.

(A)

$$L = \begin{bmatrix} \frac{7}{8} & \frac{1}{8} \\ \frac{5}{8} & \frac{3}{8} \end{bmatrix}$$

Hence $q_1 = (7/8)(1/2) + (5/8)(1/2) = 3/4$. Then $(q_1, q_2) = (3/4, 1/4)$. The potential posterior probability matrix is

$$\Pi = \begin{bmatrix} \frac{7}{12} & \frac{1}{4} \\ \frac{5}{12} & \frac{3}{4} \end{bmatrix}$$

(B) The second message service delivers more information. This is because given the first message service, either $s = 1$ or $s = 2$ has a high probability of inducing m_1. Hence receiving m_1 tells less about the actual state in the first message service.

(C)

$$\hat{L} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Given this we can compute $\hat{q} = (1/2, 1/2)$, and then we can compute the joint probability matrix and the potential posterior matrix to be

$$\hat{\Pi} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

(D) Since $\hat{\Pi}$ is invertible, we can solve $\hat{\Pi}A = \Pi$:

$$A = (\hat{\Pi})^{-1}\Pi = \begin{bmatrix} \frac{2}{3} & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$$

(E) Fix an arbitrary prior $(\pi, 1-\pi)$. It suffices to show that $\pi < \pi_{1\cdot 1} < \hat{\pi}_{1\cdot 1}$ and $\pi_{2\cdot 2} = \hat{\pi}_{2\cdot 2}$. First, we can compute q_m from the L matrices:

$$q_1 = \frac{7}{8}\pi + \frac{5}{8}(1-\pi) \quad \text{and } \hat{q}_1 = \frac{3}{4}\pi + \frac{1}{4}(1-\pi).$$

So

$$\pi_{1\cdot 1} = \frac{l_{1\cdot 1}\pi}{q_1} = \frac{\frac{7}{8}\pi}{\frac{7}{8}\pi + \frac{5}{8}(1-\pi)} = \frac{\pi}{\pi + \frac{5}{7}(1-\pi)},$$

while

$$\hat{\pi}_{1\cdot 1} = \frac{\hat{l}_{1\cdot 1}\pi}{q_1} = \frac{\frac{3}{4}\pi}{\frac{3}{4}\pi + \frac{1}{4}(1-\pi)} = \frac{\pi}{\pi + \frac{1}{3}(1-\pi)}.$$

Thus $\pi < \pi_{1\cdot 1} < \hat{\pi}_{1\cdot 1}$. To show $\pi_{2\cdot 2} = \hat{\pi}_{2\cdot 2}$, we have

$$\pi_{2\cdot 2} = \frac{\frac{3}{8}(1-\pi)}{\frac{3}{8} - \frac{2}{8}\pi} \quad \text{and } \hat{\pi}_{2\cdot 2} = \frac{\frac{3}{4}(1-\pi)}{\frac{3}{4} - \frac{2}{4}\pi}.$$

Hence they are equal. In sum, we have shown that the bracketing condition is satisfied, i.e., the agent, no matter what the prior is, becomes more certain of state 1 under $\hat{\mu}$ than under μ.

Furthermore, we can show that the Blackwell Theorem is satisfied: Since $\hat{L}$ is invertible, solving $L = \hat{L}B$ to get

$$B = \hat{L}^{-1}L = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Solution 5.2.3.4.

(A) Given $v(c) = \ln c$, the demand is given by $c_s = \pi_s W / P_s$. So

$$U(\pi) = \ln W - \pi \ln P_1 - (1-\pi)\ln P_2 + \pi \ln \pi + (1-\pi)\ln(1-\pi). \tag{2}$$

(B) Since

$$\frac{\partial U(\pi)}{\partial \pi} = -\ln P_1 + \ln P_2 + \ln \pi - \ln(1-\pi)$$

we see that

$$\frac{\partial^2 U(\pi)}{\partial^2 \pi} = \frac{1}{\pi} + \frac{1}{1-\pi} > 0$$

so $U(\pi)$ is convex.

(C) Solving

$$\begin{bmatrix} \overline{\pi}+\theta & \overline{\pi}-\theta \\ 1-\overline{\pi}-\theta & 1-\overline{\pi}+\theta \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} \overline{\pi} \\ 1-\overline{\pi} \end{bmatrix}$$

we get $q_1 = 1/2$.

Hence the gain in expected utility is given by

$$\Omega(\theta) = \frac{1}{2}U(\overline{\pi}+\theta) + \frac{1}{2}U(\overline{\pi}-\theta) - U(\pi).$$

Substituting in (2) yields the desired expression.

(D) Since

$$\frac{\partial \Omega(\theta)}{\partial \theta} = \frac{1}{2}\left(\ln(\overline{\pi}+\theta) - \ln(1-\overline{\pi}-\theta)\right) - \frac{1}{2}\left(\ln(\overline{\pi}-\theta) - \ln(1-\overline{\pi}+\theta)\right) - \ln \overline{\pi} + \ln(1-\overline{\pi}),$$

we see that $\partial\Omega(0)/\partial\theta\Big|_{\theta=0} = 0$ and by the concavity of $\ln c$, $\partial\Omega(\theta)/\partial\theta > 0$. The convexity of $\Omega(\theta)$ follows from the convexity of $U(\pi)$.

(E) Let $k(\theta)$ be the value of information. Since wealth only enters the utility by the term $\ln W$, we must have

$$\ln(\overline{W} - k(\theta)) = \ln \overline{W} - \Omega(\theta).$$

Hence

$$k(\theta) = \overline{W}(1 - e^{-\Omega(\theta)}).$$

(F) We have

$$\frac{\partial k}{\partial \theta} = \overline{W}\Omega'(\theta)e^{-\Omega(\theta)}$$

and thus

$$\frac{\partial^2 k}{\partial \theta^2} = \overline{W}[\Omega''(\theta)e^{-\Omega(\theta)} - (\Omega'(\theta))^2 e^{-\Omega(\theta)}]$$

Since $\Omega'(0) = 0$ and $\Omega''(0) > 0$, $k''(\theta) > 0$ around a neighborhood of zero. (Note that the higher derivatives of $U(\pi)$ are continuous.)

5.2.4 Differences in utility functions and the worth of information

Solution 5.2.4.1.

(A) Since the demand is $c_s = \pi_s W/P_1$ when uninformed and $c_s = W/P_s$ when informed, the willingness to pay, K, satisfies

$$\frac{1}{2}\ln(W-K) + \frac{1}{2}\ln(W-K) - \frac{1}{2}\ln(\frac{1}{2}W) - \frac{1}{2}\ln(\frac{1}{2}W).$$

Hence $K = W/2$.

(B) No matter how risk averse an agent is, the optimal consumption bundle is always riskless, both with and without conclusive information. See Figure 5.2.4.1(B), where U, V are the utilities of uninformed agents and $\hat{U}, \hat{V}$ are the utilities of the informed. The double arrows represent the worth of information – it is the same regardless of risk aversion.

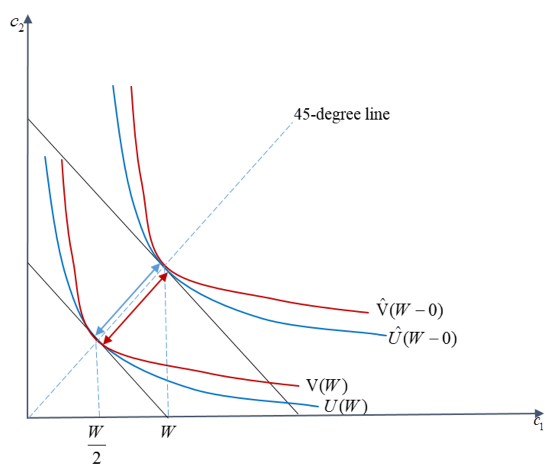

Figure 1: Ex 5.2.4.1(B)

Solution 5.2.4.2. (A) Recall that the FOC is given by

$$\frac{\pi_1 v'(c_1)}{P_1} = \frac{\pi_2 v'(c_2)}{P_2}.$$

With $\pi_2 = 1 - \pi_1$, we have

$$\frac{v'(c_2)}{v'(c_1)} = \frac{\pi_1}{1-\pi_1}\frac{P_2}{P_1} = 1.$$

(B) The optimal consumption with conclusive information is risky – see Figure 5.2.4.2. The worth of information is indicated by the the double arrow.

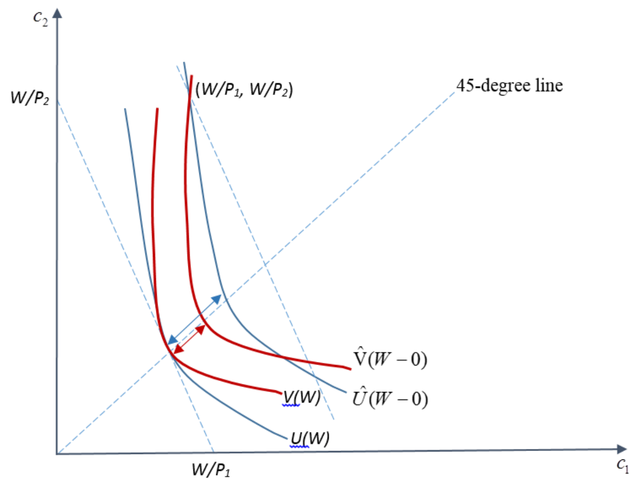

Figure 2: Ex 5.2.4.2

(C) The red utility curves in Figure 5.2.4.2 are more risk-averse than the blue utility curves. The worth of information is lower for a more risk-averse agent as the optimal consumption bundle under conclusive information is risky.

(D) It follows from the first order condition in (A) that $c_1 < c_2$. Let K be the willingness to pay for conclusive information. Then by Exercise 5.2.2.3 we have

$$K = W(1 - \pi^\pi (1-\pi)^{1-\pi}).$$

The final consumption bundle will be the lottery

$$\left(\frac{\pi^\pi (1-\pi)^{1-\pi}}{P_1}, \frac{\pi^\pi (1-\pi)^{1-\pi}}{P_2}; \pi, 1-\pi\right).$$

This is more risky than the uninformed optimal bundle

$$\left(\frac{\pi W}{P_1}, \frac{\pi W}{P_2}; \pi, 1-\pi\right)$$

if and only if

$$(1-\pi)W < W - K < \pi W$$

if and only if

$$1-\pi < \pi^{\pi}(1-\pi)^{1-\pi} < \pi$$

if and only if

$$1-\pi < \pi,$$

which is already assumed.

(E) If the price in the higher probability state is disproportionately higher, then risk aversion decreases willingness to pay for conclusive information.

Solution 5.2.4.3. Suppose after buying the information service his optimal bundle is riskier than the uninformed optimal bundle. Then being endowed with some wealth c^0 will make the DARA agent purchase the information, and make the CARA agent indifferent, and the IARA agent choose not to purchase the information.

5.2.5 The worth of information: flexibility versus range of actions

Solution 5.2.5.1.

(A) This depends on the nature of the choices x_1, x_2, x_3. Suppose x_3 is sufficiently risky, then a more risk averse agent will choose to wait. Suppose x_3 is not risky (say, riskless), then a more risk averse agent will not wait.

(B) The reasoning is similar to that of part (A) and Exercise 5.2.4.4. For example, a DARA agent with risky x_3 (relative to x_1, x_2) will now choose to take x_3.

Solution 5.2.5.2.

(A) Suppose without loss of generality the newly added action x_3 is such that $v_1(x_3) > \max\{v_1(x_1), v_1(x_2)\}$ and $v_2(x_3) < \min\{v_2(x_1), v_2(x_2)\}$. Consider two message services μ, $\hat{\mu}$ such that the bracketing condition is satisfied with $\hat{\mu}$ being more informative than μ. Suppose that message 2 implies a lower posterior for state 1, and message 1 implies a higher posterior for state 1. Then since $v_1(x_3)$ is highest, $\pi_{1\cdot 1} < \hat{\pi}_{1\cdot 1}$ will imply the expected utility under $\hat{\mu}$ is higher. This can be easily seen in Figure 5.2.5.2, where C is the expected utility without information, B is the expected utility with μ and A is the expected utility with $\hat{\mu}$.

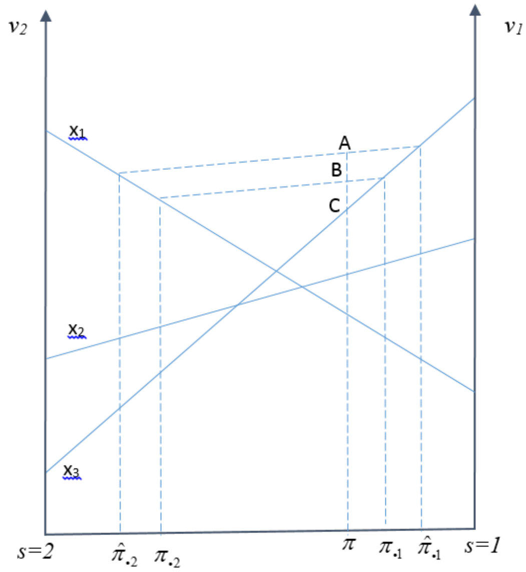

Figure 3: Ex 5.2.5.2

(B) Yes. If the information is better, it is more likely that the risky action yields a higher expected payoff. Hence, as information gets better, one is less likely to choose the riskless one.

5.3 Group Decisions

This section contains no exercises.