The Analytics of Information and Uncertainty
Answers to Exercises and Excursions

Chapter 4: Market Equilibrium Under Uncertainty

4.1 Market Equilibrium In Pure Exchange

Solution 4.1.1.

(A) It is routine to solve the demand functions to get

$$c_g^j = \frac{15(p_1 + p_2 + p_3)}{p_g}$$
$$c_g^k = \frac{1}{p_g^2} \frac{15p_1 + 67.5p_2 + 315p_3}{1/p_1 + 1/p_2 + 1/p_3}.$$

It suffices to show that the vector of prices $(p_1, p_2, p_3) = (3, 2, 1)$ clears the market. Substituting the prices into the demand function, we obtain $c_1^j = 30, c_1^k = 30$, $c_2^j = 45, c_2^k = 67.5, c_3^j = 90, c_3^k = 270$, hence the market is cleared.

(B) Let q_a^j, q_a^k be the number of units of asset a held by j, k respectively, where $q_a^j + q_a^k = 45$, and q_b^j, q_b^k such that $q_b^j + q_b^k = 1$. We need only show that there exists no $(q_a^j, q_b^j, q_a^k, q_b^k)$ that achieves the consumption vector in part (A). To this end, suppose

$$(30, 45, 90) = q_a^j(1, 1, 1) + q_b^j(15, 67.5, 315)$$
$$(30, 67.5, 270) = q_a^k(1, 1, 1) + q_b^k(15, 67.5, 315).$$

From $q_a^j + 15q_b^j = 30, q_a^j + 67.5q_b^j = 45$, we can solve $q_a^j = 25.7, q_b^j = 0.28$, it follows that $q_a^k = 34.3, q_b^k = 0.72$. But then $q_a^k + 15q_b^k = 45 \neq 30$. Hence one can not achieve the consumption in (A) by trading the two assets alone.

Solution 4.1.2. If R is constant, then preferences are homothetic. That is, for states 1 and s the Marginal Rate of Substitution $M \equiv -dc_s/dc_1$ for either individual is a function only of the probabilities and the state-consumption ratio. Also, M is equal in equilibrium to the price ratio P_1/P_s. Thus:

$$M \equiv \frac{\pi_1}{\pi_2} f(\frac{c_1^w}{c_s^w}) = \frac{P_1}{P_s} = \frac{\pi_1}{\pi_2} f(\frac{c_1^l}{c_s^l}) \equiv M^l.$$

Therefore, the equilibrium consumption ratio c_1/c_s between states 1 and s will be the same for each party. Hence, the solution must be on the main diagonal of the multi-dimensional Edgeworth box. This main diagonal is the Contract Curve. Any arrangement for proportional sharing of the state-contingent total crop will correspond to an efficient CCM equilibrium attainable from some endowment position in the box.

Solution 4.1.3.

(A) Let

$$\begin{aligned} c^w &= \omega + \gamma(y-\omega), \\ c^l &= (1-\gamma)(y-\omega). \end{aligned}$$

Then

$$\begin{aligned} \mu(c^w) &= \omega + \gamma(\mu_y - \omega), \qquad & \mu(c^l) &= (1-\gamma)(\mu_y - \omega) \\ \sigma^2(c^w) &= \gamma^2\sigma_y^2, \qquad & \sigma^2(c^l) &= (1-\gamma)^2\sigma^2(y). \end{aligned}$$

(B) The utility function is then given by

$$\begin{aligned} U^w(\omega,\gamma) &= \omega + \gamma(\mu_y - \omega) - \alpha^w\gamma^2\sigma_y^2 \\ U^l(\omega,\gamma) &= (1-\gamma)(\mu_y - \omega) - \alpha^l(1-\gamma)^2\sigma_y^2. \end{aligned}$$

Then we have

$$\frac{dU^w}{d\omega} = 1-\gamma = -\frac{dU^l}{d\omega},$$

Hence

$$\frac{dU^w}{dU^l} = -1.$$

(C) We have

$$\begin{aligned} \frac{dU^w}{d\gamma} &= -2\alpha^w\gamma\sigma_y^2 + \mu_y - \omega \\ \frac{dU^l}{d\gamma} &= 2\alpha^l(1-\gamma)\sigma_y^2 - (\mu_y - \omega). \end{aligned}$$

As $MRS^w = MRS^l$ at the optimum, it follows that

$$\frac{1-\gamma}{\gamma} = \frac{\alpha^w}{\alpha^l}.$$

Rearrange to get the desired expression.

(D) No. By (B) $(dU^w/d\omega)/(dU^l/d\omega)$ is constant, so the optimal r^* will be independent of ω.

(E) Yes. Following a similar line, one can show that $dU^w/dU^l = -M/N$. Hence by (D) the optimal γ^* is again constant.

Solution 4.1.4. To break even, the insurance company will charge $p(L + c)$, because this is the expected total payment of the insurance company.

Solution 4.1.5.

(A) The fundamental theorem of risk bearing shows

$$\frac{\pi_s A_i e^{-A_i c_s^i}}{P_s} = \frac{\pi_t A_i e^{-A c_t^i}}{P_t}$$

Hence

$$\frac{\pi_s}{\pi_t}\frac{P_t}{P_s} e^{-A_i(c_s^i - c_t^i)} = 1$$

Taking logarithms and rearrange to get

$$A_i(c_s^i - c_t^i) = \ln(\pi_s/\pi_t) - \ln(P_s/P_t). \tag{1}$$

(B) Dividing both sides of (1) by A_i and summing over i to get

$$N(\overline{c_s} - \overline{c_t}) = \sum_i \frac{1}{A_i}\left(\ln(\pi_s/\pi_t) - \ln(P_s/P_t)\right).$$

Let A^* be the harmonic mean of A_i's, then we have

$$\ln(P_s/P_t) = \ln(\pi_s/\pi_t) - A^*(\overline{c_s} - \overline{c_t}). \tag{2}$$

(C) This follows directly from (2).

(D) From (2), the price ratio will not be affected by a reallocation of the endowments. If $\overline{c_s} > \overline{c_t}$, an increase in A^* decreases P_s/P_t . If $\overline{c_s} < \overline{c_t}$, an increase in A^* increases P_s/P_t .

Solution 4.1.6. Suppose bad health happens with probability p. Then the unit price of the insurance that makes the firm break even is p dollars. That is, the insurance that pays you 1 dollar should bad health happen costs p dollars.

(A) Let $v(c, h) = \sqrt{ch}$. Let w be the endowment. If the consumer buy q units of insurance, his expected utility is then

$$U(q) = pv(w - pq + q, h_b) + (1 - p)v(w - pq, h_g).$$

The FOC with respect to q evaluated at $q = 0$ is then Hence the agent will not insure against bad health. (B) This time we have

$$\frac{dU}{dq} = p\frac{(1-p)h_b}{2\sqrt{(w/2 - pq + q)h_b}} + (1-p)\frac{-ph_g}{2\sqrt{(w-pq)h_g}}.$$

Hence

$$\left.\frac{dU}{dq}\right|_{q=0} = \frac{\sqrt{2}}{2}\frac{p(1-p)h_b}{\sqrt{wh_b}} - \frac{p(1-p)h_g}{2\sqrt{wh_g}}$$

which is larger than zero when $2h_b > h_g$. So the agent will buy some insurance when $h_g > h_b > h_g/2$. However, since

$$\left.\frac{dU}{dq}\right|_{q=w/2} = \frac{p(1-p)}{2\sqrt{w - wp/2}}(\sqrt{h_b} - \sqrt{h_g}) < 0,$$

the agent will never fully insure. (C) Let $u(x) = 2\ln(x)$. Then

$$\overline{v}(x) = u(v(c,h)) = 2\ln((ch)^{1/2}) = \ln(ch).$$

Hence $\overline{v}$ is a concave transformation of v. (D) With $\overline{v}(c,h) = \ln(ch)$, the FOC of the expected utility in the first case is given by

$$\frac{dU}{dq} = \frac{p(1-p)}{w - pq + q} - \frac{p(1-p}{w - pq}.$$

So $dU(0)/dq = 0$, not buying any insurance happens to be just optimal. In the second case,

$$\frac{dU}{dq} = \frac{p(1-p)}{w/2 - pq + q} - \frac{p(1-p}{w - pq}.$$

So $dU(w/2)/dq = 0$, the agent will fully insure. (E) For other concave transformation of $\ln(ch)$, the agent will be more risk averse, so in the first case (wealth remain the same) he will begin to insure. In the second case he will stay fully insured.

4.2 Production and Exchange

4.2.1 Equilibrium with Production: Complete Markets

Solution 4.2.1.1.

(A) First note that for arbitrary agent i,

$$\frac{\pi v'(c_1^i)}{P_1} = \frac{(1-\pi)v'(c_2^i)}{P_2}. \tag{3}$$

Then note by symmetry and convex preference we must have $c^1 = c^2 = ... = y/I$, where I is the number of agents. Plugging everything inside (3) we obtain

$$\frac{P_1}{P_2} = \frac{\pi}{1-\pi}\left(\frac{y_2}{y_1}\right) \tag{4}$$

(B) Assuming the price of riskless asset is 1, then we must have $P_1 + P_2 = 1$. Together with (4) we ca solve for P_1, P_2:

$$\begin{cases} P_1 &= \frac{\pi y_2}{\pi y_2 + (1-\pi) y_1} \\ P_2 &= \frac{(1-\pi) y_1}{\pi y_2 + (1-\pi) y_1} \end{cases} \tag{5}$$

(C) Suppose there are q_1 units of asset $z1 = (1,1)$, q^2 units of asset $z^2 = (1/2, 2)$. Then $y_1 = q_1 + 0.5q_2$, $y_2 = q_1 + 2q_2$. Plugging into (5) yields

$$P_1 = \frac{2q_1 + 4q_2}{4q_1 + 5q_2}$$
$$P_2 = \frac{2q_1 + q_2}{4q_1 + 5q_2}.$$

Then

$$P_2^A = (P_1, P_2) \cdot z^2 = \frac{5q_1 + 4q_2}{4q_1 + 5q_2}. \tag{6}$$

(D) Free entry implies price will equal marginal cost. Suppose initially there are w units of riskless asset and in equilibrium there are q_1 units of riskless asset and q_2 units of risky asset. In equilibrium we then have $P_2^A = 1$, then (6) gives $q_1 = q_2 = w/2$.

Solution 4.2.1.2.

(A) Let $y \in Y^1 + Y^2$. Then $y = (y_1^1, y_2^1) + (y_1^2, y_2^2)$, with $(y_1^i, y_2^i) \in Y^i$ for $i = 1, 2$. Hence

$$(y_1^1 + y_1^2)^2 + (y_2^1 + y_2^2)^2 \leq 4 + 2y_1^1 y_1^2 + 2y_2^1 y_2^2 \leq 8.$$

So $y \in Y$.[1]

Conversely, let $y = (y_1, y_2) \in Y$. Then $y_1^2 + y_2^2 \leq 8$. Equivalently, $(y_1/2)^2 + (y_2/2)^2 \leq 2$. Hence

$$y = \left(\frac{y_1}{2}, \frac{y_2}{2}\right) + \left(\frac{y_1}{2}, \frac{y_2}{2}\right) \in Y_1 + Y_2.$$

The firm's maximization problem is then

$$\max_{(y_1, y_2) \in Y} y_1 + y_2.$$

Hence $y_1^* = y_2^* = 2$.

(B) The agent will buy state-1 claim only since state-2 consumption does not contribute to his expected utility. The profit of the firm is $2 + 2 = 4$, hence his budget is 2. He will consume $(2, 0)$.

[1]One can verify that $\max_{(y_1^1, y_2^1) \in Y_1, (y_1^2, y_2^2) \in Y_2} 2y_1^1 y_1^2 + 2y_2^1 y_2^2 \leq 4$.

(C) Under $P = (1,1)$, the demand $(2,0),(0,2)$ maximizes each agents' expected utility, and market also clears. Hence this is an equilibrium.

(D) Yes, since the produciton of y_2 does not enter agent 1's expected value of the firm.

Solution 4.2.1.3.

(A) [graph temporarily omitted]

(B) The firm solves

$$\max_{y_1,y_2,y_3} P_{21}y_{21} + P_{22}y_{22} - P_1 y_1$$

s.t.

$$y_{21}^2 + y_{22}^2 \leq y_1.$$

The FOCs are

$$\begin{aligned} y_1: &\quad P_1 = \lambda \\ y_{21}: &\quad P_{21} = 2\lambda y_{21} \\ y_{22}: &\quad P_{22} = 2\lambda y_{22}. \end{aligned}$$

Hence

$$y_{21} = \frac{P_{21}}{2P_1}, \quad y_2 2 = \frac{P_2 2}{2P_1}, \quad y_1 = \frac{P_{21}^2 + P_{22}^2}{4P_1^2}.$$

4.2.2 Stock Market Equilibrium

Solution 4.2.2.1.

(A) Agent 1's maximization problem is

$$\max \frac{1}{3}(\ln x_1 + 3\ln y_1) + \frac{2}{3}(\ln x_2 + 3\ln y_2)$$

s.t.

$$14x_1 + 5x_2 + 10y_1 + 7y_2 \leq 24.$$

One can verify then $x_1 = 1/7, x_2 = 4/5, y_1 = 3/5, y_2 = 12/7$. Similarly, one can solve for the optimal consumption of agent 2 to be $x_1 = 6/7, x_2 = 6/5, y_1 = 2/5, y_2 = 2/7$.

(B) Stock $z^f = (1,2)$ has value $(1,2)\cdot(14,5) = 24$. Stock $z^g = (1,2)$ has value $(1,2)\cdot(10,7) = 24$.

(C) Let agent 1's portfolio be $-0.5z^f + 1.5z^g$. Suppose $s = 1$ in the second period, then his endowment will be $(x, y) = (-0.5, 1.5)$, which, when the spot price is $(P_x, P_y) = (14, 10)$, worths 8. Hence he solves in the spot market

$$\max \ln x + 3 \ln y \text{ s.t. } 14x + 10y = 8,$$

which gives $x = 1/7, y = 3/5$.

Similarly, if $s = 2$, then his endowment will be $(x, y) = (-1, 3)$, so the wealth is 16 given the spot prices $(P_x, P_y) = (5, 7)$. Hence he solves in the spot market

$$\max \ln x + 3 \ln y \text{ s.t. } 5x + 7y = 16,$$

which gives $x = 4/5, y = 12/7$.

For agent 2, in the stock market his corresponding portfolio is then $1.5z^f + (-0.5)z^g$. Again we can check the allocation proposed in (A) is his optimal consumption in the spot market in each state.

(D) By (A) and the first welfare theorem, the allocation is Pareto optimal. Hence there is no better allocation in the sense of Pareto dominance.

Solution 4.2.2.2.

(A) If $s = 1$, firm f gets $x = 1$, which has value $P_x x = 1$. Firm g gets 1, which has value $P_y y = r$. If $s = 2$, firm f gets $x = 2$, which has value $P_x x = 2$. Firm g gets 2, which has value $P_y y = 2r$. Hence in each state, the spot value of firm g is r times that of firm f, which implies the stock price of firm g should be r times that of firm f.

(B) Suppose $P_f^A = 1$. Then by (A) $P_g^A = r$. One can trade q_1 units of f-stock for q_1/r units of g-stock. In the next period with state $s \in \{1, 2\}$, the trade changes one's allocation by

$$(-q_1 + q_1/r)(s, s),$$

which has a value of zero if the spot price is $(1, r)$. Hence there is no gains from trade.

(C) With portfolio given by $0.5z^f + 0.5z^g$, in state 1 agent 1 receives $(0.5, 0.5)$, hence he solves

$$\max \ln x + 3 \ln y \text{ s.t. } x + y = 1,$$

which gives $x = 1/4, y = 3/4$. In state 2 he receives $(1, 1)$, so his income doubles. The final consumption is then $(1/2, 3/2)$. Similarly, we can solve agent 2's consumption to get $(x_1^2, x_2^2, y_1^2, y_2^2) = (3/4, 3/2, 1/4, 1/2)$.

(D) For agent 1,

$$\frac{1}{3}\left(\ln\frac{1}{7}+3\ln\frac{3}{5}\right)+\frac{2}{3}\left(\ln\frac{4}{5}+3\ln\frac{12}{7}\right)=-0.23.$$

But

$$\frac{1}{3}\left(\ln\frac{1}{4}+3\ln\frac{3}{4}\right)+\frac{2}{3}\left(\ln\frac{1}{2}+3\ln\frac{3}{2}\right)=-0.4.$$

Similarly, one can show that the consumption in Exercise 2 gives agent 2 a higher expected utility than that in (C).

4.2.3 Monopoly Power in Asset Market

Solution 4.2.3.1.

(A) Agent h solves

$$\max \pi_1^h \ln c_1^h + \pi_2^h \ln c_2^h$$

s.t.

$$P_1 c_1^h + P_2 c_2^h = \gamma^h (P_1 \overline{c_1} + P_2 \overline{c_2}).$$

The FOC is

$$\frac{\pi_1^h}{c_1^h} = P_1 \lambda, \quad \frac{\pi_2^h}{c_2^h} = P_2 \lambda,$$

plugging into the budget constraint to get $\lambda = [\gamma^h (P_1 \overline{c_1} + P_2 \overline{c_2})]^{-1}$. Hence

$$P_i c_i^h = \pi_i^h \gamma^h (P_1 \overline{c_1} + P_2 \overline{c_2}). \tag{7}$$

(B) Summing (7) over h gives $P_i \overline{c_i} = \sum_h (\pi_i^h \gamma^h)(P_1 \overline{c_1} + P_2 \overline{c_2})$ for $i = 1, 2$. Dividing the equations gives

$$\frac{P_1}{P_2} = \frac{\sum_h \pi_1^h \gamma^h}{\sum_h \pi_2^h \gamma^h} \frac{\overline{c_2}}{\overline{c_1}}. \tag{8}$$

(C) If $\pi_1^h = \pi_1$ for all h, then one can pull π_1, π_2 out from the summation and then (8) reduces to

$$\frac{P_1}{P_2} = \frac{\pi_1}{\pi_2} \frac{\overline{c_2}}{\overline{c_1}}.$$

Rearrange to get

$$\frac{\pi_1}{P_1 \overline{c_1}} = \frac{\pi_2}{P_2 \overline{c_2}}.$$

Hence

$$\frac{\pi_1 v'(\gamma^h \overline{c_1})}{P_1} = \frac{\pi_2 v'(\gamma^h \overline{c_2})}{P_2}, \tag{9}$$

which means the endowment is already optimal.

(D) Suppose there is no trade. Then the endowment will satisfy (9) for all h. That is,

$$\frac{\pi_1^h}{\pi_2^h} = \frac{P_1 \gamma^h \overline{c_1}}{P_2 \gamma^h \overline{c_2}} = \frac{P_1 \overline{c_1}}{P_2 \overline{P_2}}.$$

Hence everyone has the same belief. So the converse holds.

(E) Yes, since the commodity space is 2-dimensional, and any two linearly independent vectors span the space.

Solution 4.2.3.2.

(A) Suppose there are $\overline{q_a}$ units of asset a and $\overline{q_b}$ units of asset b. Then the endowment is $\overline{c_1} = \overline{q_a} + \beta\overline{q_b}, \overline{c_2} = \alpha\overline{q_a} + \overline{q_b}$. By (8),

$$\frac{P_1}{P_2} = \frac{\sum_h \pi_1^h \gamma^h}{\sum_h \pi_2^h \gamma^h} \frac{\overline{q_a} + \beta\overline{q_b}}{\alpha\overline{q_a} + \overline{q_b}}.$$

(B) Suppose $P_1 = \sum_h (\pi_1^h \gamma^h)(\overline{q_a} + \beta\overline{q_b})k$ and $P_2 = \sum_h (\pi_2^h \gamma^h)(\alpha\overline{q_a} + \overline{q_b})k$. Then

$$\begin{aligned} P_a^A = (P_1, P_2) \cdot z^\alpha &= \left((\sum_h \pi_1^h \gamma^h)(\overline{q_a} + \beta\overline{q_b}) + \alpha(\sum_h \pi_2^h \gamma^h)(\alpha\overline{q_a} + \overline{q_b}) \right) k \\ P_b^A = (P_1, P_2) \cdot z^\beta &= \left(\beta(\sum_h \pi_1^h \gamma^h)(\overline{q_a} + \beta\overline{q_b}) + (\sum_h \pi_2^h \gamma^h)(\alpha\overline{q_a} + \overline{q_b}) \right) k. \end{aligned}$$

The price ratio is then

$$\frac{P_a^A}{P_b^A} = \frac{\overline{q_a} + \beta\overline{q_b} + \alpha(\overline{q_b} + \alpha\overline{q_a})}{\beta(\overline{q_a} + \beta\overline{q_b}) + (\overline{q_b} + \alpha\overline{q_a})} \frac{\pi_1}{\pi_2} \tag{10}$$

(C) From (10), it must be that $\alpha = \beta = 1$.

(D) Free entry implies q_b is such that $P_a^A / P_b^A = 1$. By (9) and $q_a + q_b = 1000$, we will have $q_a = q_b = 500$.

4.3 Asset Prices in the μ, σ model

Solution 4.3.1.

(A) Yes. Given convex preference and everyone is alike, the endowment $(1, 1, 1)$ must be optimal. The price of risk reduction is then the MRS at that point, given by $\mu(z_1 + z_2 + z_3) = 3, \sigma(z_1 + z_2 + z_3) = 5$. Since $U(\mu, \sigma) = \mu^{10} e^{-\sigma}$, the MRS is then

$$-\frac{dU/d\sigma}{dU/d\mu} = \frac{e^{-5} 3^{10}}{10 \cdot 3^9 e^{-5}} = \frac{3}{10}.$$

(B) Let $(1, P_2^A, P_3^A)$ be the equilibrium prices. The maximization problem is given by

$$\max_{q_1,q_2,q_3} (q_1+q_2+q_3)^{10} e^{-\sqrt{9q_2^2+16q_3^2}}.$$

s.t.

$$q_1 + P_2^A q_2 + P_3^A q_3 = 1 + P_2^A + P_3^A.$$

The FOCs are

$$\begin{aligned}
q_1:&\quad 10(q_1+q_2+q_3)^9 e^{-\sqrt{9q_2^2+16q_3^2}} = \lambda.\\
q_2:&\quad 10(q_1+q_2+q_3)^9 e^{-\sqrt{9q_2^2+16q_3^2}} - \frac{1}{10}10(q_1+q_2+q_3)^9 e^{-\sqrt{9q_2^2+16q_3^2}} \frac{18q_2}{2\sqrt{9q_2^2+16q_3^2}} = \lambda P_2^A\\
q_3:&\quad 10(q_1+q_2+q_3)^9 e^{-\sqrt{9q_2^2+16q_3^2}} - \frac{1}{10}10(q_1+q_2+q_3)^9 e^{-\sqrt{9q_2^2+16q_3^2}} \frac{32q_3}{2\sqrt{9q_2^2+16q_3^2}} = \lambda P_3^A.
\end{aligned}$$

By (A), $(q_1, q_2, q_3) = (1, 1, 1)$ solves the above equations. Plugging in, we see that

$$1 - \frac{3}{10}\frac{18}{10} = P_2^A \qquad\qquad 1 - \frac{3}{10}\frac{32}{10} = P_3^A.$$

Hence the prices given in Exercise 2.2.4 is an equilibrium price. Let $\tilde{R}_i$ be the single asset portfolio on z^i, then $E[\tilde{R}_2] = 3.26$, $\sigma(\tilde{R}_2) = 9.78$. $\mu(\tilde{R}_3) = 37.5$, $\sigma(\tilde{R}_3) = 150$.

(C) Since $\theta = 1/3$ and $R_1 = 0$ the security valuation line (I) is given by

$$\mu(\tilde{R}_a) = \frac{3}{10}\rho(\tilde{R}_a, \tilde{R}_f)\sigma(\tilde{R}_a).$$

The riskless portfolio corresponds to the point $(0, 0)$, single asset portfolio of risky asset 1 corrsponds to the point $(10.86, 3.26)$. And so on.

(D) The security valuation line is given by

$$\mu(\tilde{R}_a) = \frac{3}{10}\cdot 15 \cdot \beta_a = 9\beta_a.$$

Since we know $\mu(\tilde{R}_a)$ for each a, we can get the corresponding β_a easily.

Solution 4.3.2.

(A) Assume there are N identical agents with budget w. Let the mutual fund be z^F, the riskless asset be z_1. Assume in optimum each agent consumes $1/N$ units of riskless asset and $1/N$ units of risky asset. Then as N grows large, $\sigma(1/N + z^F/N) \to 0$. Hence the marginal utility to σ becomes 0. The price of risk reduction in equilibrium equals

$$-\frac{\partial U/\partial\sigma}{\partial U/\partial\mu} = 0$$

(B) Let z^n denote the risky asset given by firm n, where $n \leq kN$. The asset holding of an individual will then be

$$\frac{1}{N}z_1 + \frac{1}{N}z_2 + ... + \frac{1}{N}z^{kN}.$$

Then the variance is given by

$$\sigma^2 = \frac{kN\sigma^2}{N^2} \to 0$$

as $N \to \infty$. Hence by (A) the price of risk reduction goes to zero.

(C) If the population is large, the insurance company will be able to cover independent risks. However, the insurance company is not able to do so against earthquakes in LA, since in such cases agents' risks are perfectly correlated instead of independent.

Solution 4.3.3.

(A) If the individual purchases q units of risky asset, then his portfolio is $x = (W - P_F q) + q\tilde{z}_F$. Since $\mu(x) = W - P_F q + q\mu_F$ and $\sigma(x) = q\sigma_F$, his utility is then

$$(W_j - P_F q)(1 + R) + q\mu_F - \frac{1}{2}\alpha_j q^2 \sigma_F^2.$$

(B) The individual solves

$$\max_q (W_j - P_F q)(1 + R) + q\mu_F - \frac{1}{2}\alpha_j q^2 \sigma_F^2$$

The first order condition then gives

$$q^j = \frac{\mu_F - P_F(1 + R)}{\alpha_j \sigma_F^2} \tag{11}$$

(C) Summing (11) over j and apply the market clearing condition $\sum_j q_j = 1$ to obtain

$$\frac{\mu_F - P_F(1 + R)}{\sigma_F^2} \sum_j \frac{1}{\alpha_j} = 1.$$

Hence

$$P_F = \frac{1}{1 + R}\left(\mu_F - \frac{\overline{\alpha}}{J}\sigma_F^2\right). \tag{12}$$

(D) This follows from (12) and the CAPM rule (4.3.1).

(E) This follows from (D) and (4.3.1).

(F) Suppose asset a is uncorrelated to other assets, then

$$\sigma_{aF} = \sigma_a^2.$$

Plugging into the expression in (E) and rearrange gets the desired expression.

Solution 4.3.4.

(A) Pick an arbitrary asset a. The mutual fund F is $\sum_{a \in A} z_a$. Hence

$$\sigma_{aF} = E[(z_a - \mu_a)(\sum a' \in A z_{a'})] = ((A-1)\rho + 1)\sigma^2.$$

The claim then follows from 4.3.4(E).

(B) The mutual fund is now $\sum_{a' \neq a} z_{a'} + q z_a$. Hence

$$\sigma a F = (A-1)\rho\sigma^2 + q\sigma^2.$$

It again follows from 4.3.4(E) that

$$P_a^A = \frac{1}{1+R}\left(\mu - \frac{\overline{\alpha}}{J}((A-1)\rho + q)\sigma^2\right). \tag{13}$$

(C) The profit maximizing q^* satisfies $MR = MC$, where $\Pi(q) = qP_a^A(q) - c(q)$. But the efficient supply q^e satisfies $c'(q) = P_a^A(q)$. Monopoly production level is then less than the efficient level.

(D) When J and A grows proportionally large, by (12) the effect of q to $P_a^A(q)$ approaches 0, hence the demand function becomes more flat. In this case the monopoly power becomes smaller.